Chapter 14

Trigonometric Ratios and Identities

Exercise

1. The value of $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$ is
   (a) 1 (b) 2
   (c) 3 (d) 6
2. $(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$ is equal to
   (a) 0 (b) 1
   (c) – 1 (d) None of these
3. $\left(\frac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}\right)^2$ is equal to
   (a) $\frac{1-\sin\theta}{1+\sin\theta}$ (b) $\frac{1+\sin\theta}{1-\sin\theta}$
   (c) 1 (d) None of these
4. If $\pi \le \alpha < \frac{3\pi}{2}$, then $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} + \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$ is equal to
   (a) 2 cosec α (b) – 2 cosec α
   (c) 2 sec α (d) – 2 sec α
5. If $\cos\theta = -\frac{1}{2}$ and θ lies in the second quadrant, then the value of (2 sin θ + tan θ)is
   (a) 0 (b) $-\frac{\sqrt{3}}{2}$
   (c) $\frac{3\sqrt{3}}{2}$ (d) None of these
6. The value of (cos 24° + cos 55° + cos 125° + cos 204° + cos 300°) is
   (a) 0 (b) $\frac{3}{2}$
   (c) 3 (d) $\frac{1}{2}$
7. If $x = h + p\sec\alpha$ and $y = k + q\,\text{cosec}\,\alpha$, then
   (a) $\frac{(x-h)^2}{p^2} + \frac{(y-k)^2}{q^2} = 1$
   (b) $\frac{(x-h)^2}{p^2} - \frac{(y-k)^2}{q^2} = 1$
   (c) $\frac{p^2}{(x-h)^2} + \frac{q^2}{(y-k)^2} = 1$
   (d) $\frac{p^2}{(x-h)^2} - \frac{q^2}{(y-k)^2} = 1$
8. The value of (tan 20° + tan 40° + tan 60° + ... + tan 180°) is
   (a) 9 (b) $4\frac{1}{2}$
   (c) 0 (d) 1
9. The value of (sin 10° + sin 20° + sin 30° + ... + sin 360°) is
   (a) 0 (b) 1
   (c) – 1 (d) None of these
10. The value of ($\sin^2 5° + \sin^2 10° + \sin^2 15°$ + ... + $\sin^2 85° + \sin^2 90°$) is
    (a) 7 (b) 8
    (c) 9 (d) $9\frac{1}{2}$
11. The value of tan 75° – tan 30° – tan 75° tan 30° is
    (a) 1 (b) 0
    (c) $\frac{3}{2}$ (d) 2
12. ($\sqrt{3}$ cosec 20° – sec 20°) is equal to
    (a) 2 (b) 4
    (c) 5 (d) 7
13. (tan 58°– tan 13° – tan 58° tan 13°) is equal to
    (a) – 2 (b) 2
    (c) – 1 (d) 1
14. The value of ($2\cos\alpha - \cos 3\alpha - \cos 5\alpha - 16\cos^3\alpha\sin^2\alpha$) is

(a) 2 (b) 1
(c) 0 (d) – 1

15. If α and β are acute angles such that $\tan \alpha = \frac{1}{7}$ and $\sin \beta = \frac{1}{\sqrt{10}}$ then (α + 2β) is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{12}$

16. If (α + β) = 90°, the minimum and maximum values of (cos α cos β) are respectively

(a) $-\frac{1}{4}$ and $\frac{1}{4}$ (b) $-\frac{1}{3}$ and $\frac{1}{3}$
(c) $-\frac{1}{2}$ and $\frac{1}{2}$ (d) None of these

17. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x$ is equal to

(a) 0 (b) –1
(c) 2 (d) 1

18. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x + 2\cos^4 x + \cos^2 x - 2$ is equal to

(a) 0 (b) 1
(c) 2 (d) $\sin^2 x$

19. If $\cos A = \frac{3}{4}$, then $32 \sin\frac{A}{2} \sin\frac{5A}{2}$ is equal to

(a) 7 (b) 8
(c) 11 (d) None of these

20. $\frac{\sec 8\theta - 1}{\sec 4\theta - 1}$ equal to

(a) tan 8θ tan 2θ (b) tan 8θ / tan 2θ
(c) cot 8θ cot 2θ (d) None of these

21. If cot (α + β) = 0, then sin (α + 2β) is

(a) – sin α (b) sin β
(c) cos α (d) cos β

22. Which is larger, sin 26° or cos 26°?

(a) sin26° (b) cos26°
(c) Both are equal (d) cannot be compared

23. If tan *A* + tan *B* + tan *C* = tan *A* tan *B* tan *C*, then *A* + *B* + *C* is equal to

(a) π (b) 2π
(c) $\frac{\pi}{2}$ (d) None of these

24. $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{2}{2}$
(c) $\frac{3}{2}$ (d) $\frac{4}{2}$

25. If sin *A* + sin *B* = *a* and cos *A* + cos *B* = *b*, then cos (*A* + *B*) is

(a) $\frac{a^2+b^2}{b^2-a^2}$ (b) $\frac{2ab}{a^2+b^2}$
(c) $\frac{b^2-a^2}{a^2+b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

26. If $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$, then tan *A*, tan *B*, tan *C* are in

(a) AP (b) GP
(c) HP (d) None of these

27. If $\tan^2\theta = 2\tan^2\phi + 1$, then $\cos 2\theta + \sin^2\phi$ is

(a) 1 (b) 2
(c) 0 (d) – 1

28. The value of $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{16}$
(c) $\frac{1}{64}$ (d) None of these

29. $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$ is equal to

(a) $1/2^6$ (b) $1/2^7$
(c) $1/2^8$ (d) None of these

30. If *A* = tan 6° tan 42° and *B* = cot 66° cot 78°, then

(a) *A* = 2*B* (b) $A = \frac{1}{B}$
(c) *A* = *B* (d) 3*A* = 2*B*

31. If sin(α + β) = 1, sin(α – β) = $\frac{1}{2}$, α, β ∈ [0, $\frac{\pi}{2}$], then tan (a + 2β) tan (2α + β) is equal to

(a) 1 (b) 2
(c) – 1 (d) None of these

32. If $m = \sin^6 x + \cos^6 x$, then *m* belongs to the interval

(a) $\left[\frac{7}{8}, \frac{5}{4}\right]$ (b) $\left[\frac{1}{2}, \frac{5}{8}\right]$
(c) $\left[\frac{1}{4}, 1\right]$ (d) None of these

33. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ is

(a) 9 (b) 10
(c) 13 (d) 6

34. Let *f* (θ) = sin θ (sin θ + sin 3θ). Then, *f* (θ)

(a) ≥ 0 only when θ ≥ 0 (b) ≤ 0 for all real θ
(c) ≥ 0 for all real θ (d) ≤ 0 only when θ ≤ 0

35. If sin *A* + cos *A* = *p*, $\sin^3 A + \cos^3 A = q$, then

(a) $p^3 - 3p + q = 0$

(b) $q^3 - 3q + p = 0$
(c) $p^3 - 3p + 2q = 0$
(d) $q^3 - 3q + 2p = 0$

36. If $\sin(x + 3\alpha) = 3\sin(\alpha - x)$ then
(a) $\tan x = \tan\alpha$ (b) $\tan x = \tan^2\alpha$
(c) $\tan x = \tan^3\alpha$ (d) $\tan x = 3\tan\alpha$

37. The value of $\sin^3 10° + \sin^3 50° - \sin^3 70°$ is
(a) $-\frac{3}{2}$ (b) $\frac{3}{4}$
(c) $-\frac{3}{4}$ (d) $-\frac{3}{8}$

38. If $A = \sin 45° + \cos 45°$ and $B = \sin 44° + \cos 44°$, then
(a) $A > B$ (b) $A < B$
(c) $A = B$ (d) None of these

39. If $\sin x = \frac{1}{2}$ and x lies in the second quadrant, then the value of $\cos\left(\frac{x}{2}\right)$ is
(a) $\frac{\sqrt{2-\sqrt{3}}}{2}$ (b) $\frac{\sqrt{2+\sqrt{3}}}{2}$
(c) $\frac{\sqrt{(\sqrt{2}+1)}}{2}$ (d) $\frac{\sqrt{(\sqrt{3}-1)}}{2}$

40. $\frac{\sin 50° - \sin 40°}{\cos 50° - \cos 40°}$ is equal to
(a) -2 (b) 1
(c) -1 (d) 2

41. The value of $(\cos 10° - \sin 10°)$ is
(a) positive (b) negative
(c) zero (d) None of these

42. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $(m^2 - n^2)$ is
(a) $4mm$ (b) $4\sqrt{mn}$
(c) $2mn$ (d) $2\sqrt{mn}$

43. If $\sin\alpha + \operatorname{cosec}\alpha = 2$, then the value of $(\sin^2\alpha + \operatorname{cosec}^2\alpha)$ is
(a) 1 (b) 2
(c) 4 (d) None of these

44. $\frac{\sin A + \sin 3A}{\cos A + \cos 3A}$ is equal to
(a) $\tan A$ (b) $\tan 2A$
(c) $\cot 2A$ (d) $\cot A$

45. The value of $\left(\tan\frac{\pi}{20}\tan\frac{3\pi}{20}\tan\frac{5\pi}{20}\tan\frac{7\pi}{20}\tan\frac{9\pi}{20}\right)$ is
(a) -1 (b) 1
(c) $\frac{1}{2}$ (d) ∞

46. If $\theta = \frac{\pi}{4n}$, then the value of
$\tan\theta\tan 2\theta \ldots\ldots \tan(2n-2)\theta\tan(2n-1)\theta$ is
(a) -1 (b) 1
(c) 0 (d) 2

47. $\tan 20° + 2\tan 50° - \tan 70°$ is equal to
(a) 1 (b) 0
(c) $\tan 50°$ (d) None of these

Directions (Q. 48 to 50) :

Consider the following $f(\theta) = 4(\sin^2\theta + \cos^2\theta)$

48. What is the maximum value of the function $f(\theta)$? **[NDA-I 2016]**
(a) 1 (b) 2
(c) 3 (d) 4

49. What is the minimum value of the function $f(\theta)$? **[NDA-I 2016]**
(a) 0 (b) 1
(c) 2 (d) 3

50. Consider the following statements **[NDA-I 2016]**
1. $f(\theta) = 2$ has no solution
2. $f(\theta) = \frac{7}{2}$ has a solution

Which of the above statement(s) is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

51. If $A = (\cos 12° - \cos 36°)(\sin 96° + \sin 24°)$ and $B = (\sin 60° - \sin 12°)(\cos 48° - \cos 72°)$, then what is $\frac{A}{B}$ equal to? **[NDA-I 2016]**
(a) -1 (b) 0
(c) 1 (d) 2

Directions (Q. 52 to 53) :

Given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$

52. What is $\tan(\alpha + \beta)$ equal to? **[NDA-I 2016]**
(a) $b(c-1)$ (b) $c(b-1)$
(c) $c(b-1)^{-1}$ (d) $b(c-1)^{-1}$

53. What is $\sin(\alpha + \beta)\sec\alpha\sec\beta$ equal to? **[NDA-I 2016]**
(a) b (b) $-b$
(c) c (d) $-c$

Directions (Q. 54 to 55) :

Consider a triangle ABC in which

$$\cos A + \cos B + \cos C = \sqrt{3}\sin\frac{\pi}{3}$$

54. What is the value of $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$? **[NDA-I 2016]**
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

55. Value of $\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)$ is **[NDA-I 2016]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{16}$ (d) None of these

56. Consider the following statements

1. If ABC is an equilateral triangle, then $3\tan(A+B)\tan C = 1$
2. If ABC is a triangle in which $A = 78°$, $B = 66°$, then $\tan\left(\frac{A}{2}+C\right) < \tan A$
3. If ABC is any triangle, then $\tan\left(\frac{A+B}{2}\right)\sin\left(\frac{C}{2}\right) < \cos\left(\frac{C}{2}\right)$ **[NDA-I 2016]**

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2

(c) 1 and 2 (d) 2 and 3

Directions (Q. 57 to 58) :

Consider the equation $k\sin x + \cos 2x = 2k - 7$

57. If the equation possesses solution, then what is the minimum value of k? **[NDA-I 2016]**

(a) 1 (b) 2

(c) 4 (d) 6

58. If the equation possesses solution, then what is the maximum value of k? **[NDA-I 2016]**

(a) 1 (b) 2

(c) 4 (d) 6

59. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is equal to **[NDA-II2016]**

(a) – 1 (b) 0

(c) 1 (d) 4

60. If $a\cos\theta + b\sin\theta = c$, then $(a\sin\theta - b\cos\theta)^2$ is **[NDA-II 2016]**

(a) $a^2 + b^2 - c^2$ (b) $a^2 - b^2 - c^2$

(c) $a^2 - b^2 + c^2$ (d) $a^2 + b^2 + c^2$

61. If $\frac{(1-\tan 2°\cot 62°)}{(\tan 152° - \cot 88°)} = k\sqrt{3}$, then the value of k is **[NDA-II 2016]**

(a) $-\frac{1}{2}$ (b) – 1

(c) 1 (d) $\frac{1}{2}$

62. Find the value of

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right).$$

[NDA-II 2016]

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$

(c) $\frac{1}{4}$ (d) None of these

63. $\sin A + 2\sin 2A + \sin 3A$ is equal to which of the following. **[NDA-II2016]**

1. $4\sin 2A\cos^2\left(\frac{A}{2}\right)$
2. $2\sin 2A\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)^2$
3. $8\sin A\cos A\cos^2\left(\frac{A}{2}\right)$

Select the correct answer using the code given below

(a) 1 and 2

(b) Only 2 and 3

(c) Only 1 and 3 only

(d) 1, 2 and 3

64. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4 = 4$, then what is the value of $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 + \cos\theta_4$? **[NDA-II2016]**

(a) 0 (b) 1

(c) 2 (d) 4

65. If $\sin A = \frac{3}{5}$, where $450° < A < 540°$, then $\cos\frac{A}{2}$ is equal to **[NDA-II2016]**

(a) $\frac{1}{\sqrt{10}}$ (b) $-\sqrt{\frac{3}{10}}$

(c) $\sqrt{\frac{3}{10}}$ (d) None of these

66. What is $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°}$ equal to **[NDA-I 2017]**

(a) 0 (b) 2

(c) 1 (d) 4

67. If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta + \alpha) + 2\tan\alpha$ is equal to **[NDA-I 2017]**

(a) – 1 (b) 0

(c) 1 (d) 2

68. If $\tan(\alpha + \beta) = 2$ and $\tan(\alpha - \beta) = 1$, then $\tan(2\alpha)$ is equal to **[NDA-I 2017]**

(a) – 3 (b) – 2

(c) $-\frac{1}{3}$ (d) 1

69. The expression $\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta}$ is equal to **[NDA-I 2017]**

(a) $\tan\left(\frac{\alpha+\beta}{2}\right)$ (b) $\cot\left(\frac{\alpha+\beta}{2}\right)$

(c) $\sin\left(\frac{\alpha+\beta}{2}\right)$ (d) $\cos\left(\frac{\alpha+\beta}{2}\right)$

70. Consider the following for ΔABC.

1. $\sin\left(\frac{B+C}{2}\right) = \cos\left(\frac{A}{2}\right)$
2. $\tan\left(\frac{B+C}{2}\right) = \cot\left(\frac{A}{2}\right)$
3. $\sin(B + C) = \cos A$
4. $\tan(B + C) = -\cot A$

Which of the above are correct? **[NDA-I 2017]**

(a) 1 and 3 (b) 1 and 2
(c) 1 and 4 (d) 2 and 3

71. If $\sec\theta - \text{cosec}\,\theta = \frac{4}{3}$, then what is $(\sin\theta - \cos\theta)$ equal to ? **[NDA-I 2017]**

(a) Only -2 (b) Only $\frac{1}{2}$
(c) Both -2 and $\frac{1}{2}$ (d) Neither $\frac{1}{2}$ nor -2

72. The minimum value of $3\sin\theta + 4\cos\theta$ is **[NDA-II2017]**

(a) 1 (b) 3
(c) -5 (d) 5

73. In ΔABC, if $\frac{\sin^2 A+\sin^2 B+\sin^2 C}{\cos^2 A+\cos^2 B+\cos^2 C} = 2$ then the triangle is **[NDA-II2017]**

(a) right angled (b) equilateral
(c) isosceles (d) obtuse angled

74. In ΔPQR, $\angle R = \frac{\pi}{2}$, if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct ? **[NDA-II2017]**

(a) $a = b + c$ (b) $b = c + a$
(c) $c = a + b$ (d) $b = c$

75. Angle α is divided into two parts A and B such that $A - B = x$ and $\tan A : \tan B = p : q$. The value of $\sin x$ is equal to **[NDA-II2017]**

(a) $\frac{(p+q)\sin\alpha}{p-q}$ (b) $\frac{p\sin\alpha}{p+q}$
(c) $\frac{p\sin\alpha}{p-q}$ (d) $\frac{(p-q)\sin\alpha}{p+q}$

76. $\sqrt{1+\sin A} = -\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)$ is true if **[NDA-II2017]**

(a) $\frac{3\pi}{2} < A < \frac{5\pi}{2}$ (b) $\frac{\pi}{2} < A < \frac{3\pi}{2}$
(c) $\frac{3\pi}{2} < A < \frac{7\pi}{2}$ (d) $0 < A < \frac{3\pi}{2}$

77. If $x = \sin 70° \sin 50°$ and $y = \cos 60° \cos 80°$, then what is xy equal to? **[NDA-II 2017]**

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

78. If $\sin x = \frac{1}{\sqrt{5}}$, $\sin y = \frac{1}{\sqrt{10}}$, where $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, then what is $(x + y)$ equal to? **[NDA-II 2018]**

(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) 0

79. What is $\frac{\sin 5x-\sin 3x}{\cos 5x+\cos 3x}$ equal to **[NDA-I 2018]**

(a) $\sin x$ (b) $\cos x$
(c) $\tan x$ (d) $\cot x$

80. What is $\sin 105° + \cos 105°$ equal to **[NDA-I 2018]**

(a) $\sin 50°$ (b) $\cos 50°$
(c) $\frac{1}{\sqrt{2}}$ (d) 0

81. If x, $x - y$ and $x + y$ are the angles of a triangle (not an equilateral triangle) such that $\tan(x - y)$, $\tan x$ and $\tan(x + y)$ are in GP, then what is x equal to? **[NDA-I 2018]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

82. ABC is a triangle inscribed in a circle with centre O. Let $\alpha = \angle BAC$, where $45° < \alpha < 90°$. Let $\beta = \angle BOC$. Which one of the following correct ? **[NDA-I 2018]**

(a) $\cos\beta = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ (b) $\cos\beta = \frac{1+\tan^2\alpha}{1-\tan^2\alpha}$
(c) $\cos\beta = \frac{2\tan\alpha}{1+\tan^2\alpha}$ (d) $\sin\beta = 2\sin^2\alpha$

83. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then what is $\frac{\tan x}{\tan y}$ equal to? **[NDA-I 2018]**

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$
(c) $\frac{a+b}{a-b}$ (d) $\frac{a-b}{a+b}$

84. If $\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$, where $0 < \beta < \alpha < 2\pi$, then which one of the following is correct? **[NDA-I 2018]**

(a) $\alpha = \pi - \beta$ (b) $\alpha = \pi + \beta$
(c) $\alpha = 2\pi - \beta$ (d) $2\alpha = \pi + 2\beta$

85. Suppose cos A is given. If only one value of $\cos\left(\frac{A}{2}\right)$ is possible, then A must be **[NDA-I 2018]**
(a) an odd multiple of 90°
(b) a multiple of 90°
(c) an odd multiple of 180°
(d) a multiple of 180°

86. If $\cos\alpha + \cos\beta + \cos\gamma = 0$, where $0 < \alpha \le \frac{\pi}{2}, 0 < \beta \le \frac{\pi}{2}, 0 < \gamma \le \frac{\pi}{2}$, then what is the value of $\sin\alpha + \sin\beta + \sin\gamma$? **[NDA-I 2018]**
(a) 0 (b) 3
(c) $\frac{5\sqrt{2}}{2}$ (d) $\frac{3\sqrt{2}}{2}$

87. The maximum value of $\sin\left(x+\frac{\pi}{5}\right)+\cos\left(x+\frac{\pi}{5}\right)$, where $x \in \left(0, \frac{\pi}{2}\right)$, is attained at **[NDA-I 2018]**
(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{15}$
(c) $\frac{\pi}{10}$ (d) $\frac{\pi}{2}$

88. What is the maximum value of $16\sin\theta - 12\sin^2\theta$? **[NDA-I 2018]**
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{16}{3}$ (d) 4

89. If $A + B + C = 180°$, then what is $\sin 2A - \sin 2B - \sin 2C$ equal to ? **[NDA-I 2018]**
(a) $-4\sin A\sin B\sin C$ (b) $-4\cos A\sin B\cos C$
(c) $-4\cos A\cos B\sin C$ (d) $-4\sin A\cos B\cos C$

90. A is an angle in the fourth quadrant. It satisfies the trigonometric equation $3(3-\tan^2 A-\cot A)^2 = 1$. Which one of the following is the value of A ? **[NDA-II2018]**
(a) 300º (b) 315º
(c) 330º (d) 345º

91. What is/are the solution(s) of the trigonometric equation $\text{cosec}\,x + \cot x = \sqrt{3}$, where $0 < x < 2\pi$? **[NDA-II 2018]**
(a) Only $\frac{5\pi}{3}$ (b) Only $\frac{\pi}{3}$
(c) Only π (d) π, $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

92. What is $\frac{2\tan\theta}{1+\tan^2\theta}$ equal to? **[NDA-II2018]**
(a) cos 2θ (b) tan 2θ
(c) sin 2θ (d) cosec 2θ

93. If $\theta = \frac{\pi}{8}$, then what is the value of $(2\cos\theta + 1)^{10}$ $(2\cos 2\theta - 1)^{10}$ $(2\cos\theta - 1)^{10}(2\cos 4\theta - 1)^{10}$? **[NDA-II 2018]**
(a) 0 (b) 1
(c) 2 (d) 4

94. If cos α and cos β $(0 < \alpha < \beta < \pi)$ are the roots of the quadratic equation $4x^2 - 3 = 0$, then what is the value of $\sec\alpha \times \sec\beta$? **[NDA-II2018]**
(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$
(c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

95. If sin β is the harmonic mean of sin α and cos α and sin θ is the arithmetic mean of sin α and cos α, then which of the following is/are correct ?
1. $\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)\sin\beta = \sin 2\alpha$
2. $\sqrt{2}\sin\theta = \cos\left(\alpha-\frac{\pi}{4}\right)$

Select the correct answer using the code given below. **[NDA-II2018]**
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

96. If $m\tan(\theta - 30°) = n\tan(\theta + 120°)$, then $\frac{m+n}{m-n}$ is equal to **[NDA-II2018]**
(a) sin 2θ (b) 2cos 2θ
(c) tan 2θ (d) None of these

97. Given $A = \sin^2\theta + \cos^4\theta$ then for all real θ, **[NDA-II 2018]**
(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$
(c) $\frac{13}{16} \le A \le 1$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

98. tan 54° can be expressed as **[NDA-I 2019]**
(a) $\frac{\sin 9° + \cos 9°}{\sin 9° - \cos 9°}$ (b) $\frac{\sin 9° - \cos 9°}{\sin 9° + \cos 9°}$
(c) $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$ (d) $\frac{\sin 36°}{\cos 36°}$

99. If sin 2θ = cos3θ and θ is an acute angle, then sin θ equals to **[NDA-I 2019]**
(a) $\frac{\sqrt{5}-1}{4}$ (b) $-\left(\frac{\sqrt{5}-1}{4}\right)$
(c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{-\sqrt{5}-1}{4}$

100. What is the value of $\frac{\sin 34°\cos 236° - \sin 56°\sin 124°}{\cos 28°\cos 88° + \cos 178°\sin 208°}$? **[NDA-I 2019]**
(a) – 2 (b) – 1
(c) 2 (d) 1

ANSWERS

1.	(a)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(a)	**6.**	(d)	**7.**	(c)	**8.**	(c)	**9.**	(a)	**10.**	(d)
11.	(a)	**12.**	(b)	**13.**	(d)	**14.**	(c)	**15.**	(b)	**16.**	(c)	**17.**	(d)	**18.**	(d)	**19.**	(c)	**20.**	(b)
21.	(d)	**22.**	(b)	**23.**	(a)	**24.**	(c)	**25.**	(c)	**26.**	(b)	**27.**	(c)	**28.**	(b)	**29.**	(b)	**30.**	(c)
31.	(a)	**32.**	(c)	**33.**	(c)	**34.**	(c)	**35.**	(c)	**36.**	(c)	**37.**	(d)	**38.**	(a)	**39.**	(a)	**40.**	(c)
41.	(a)	**42.**	(b)	**43.**	(b)	**44.**	(b)	**45.**	(b)	**46.**	(b)	**47.**	(b)	**48.**	(d)	**49.**	(d)	**50.**	(b)
51.	(c)	**52.**	(d)	**53.**	(b)	**54.**	(d)	**55.**	(d)	**56.**	(b)	**57.**	(b)	**58.**	(d)	**59.**	(d)	**60.**	(a)
61.	(b)	**62.**	(a)	**63.**	(c)	**64.**	(a)	**65.**	(d)	**66.**	(d)	**67.**	(b)	**68.**	(a)	**69.**	(a)	**70.**	(b)
71.	(b)	**72.**	(c)	**73.**	(a)	**74.**	(c)	**75.**	(d)	**76.**	(c)	**77.**	(a)	**78.**	(c)	**79.**	(c)	**80.**	(c)
81.	(b)	**82.**	(a)	**83.**	(a)	**84.**	(b)	**85.**	(c)	**86.**	(b)	**87.**	(a)	**88.**	(c)	**89.**	(d)	**90.**	(a)
91.	(b)	**92.**	(c)	**93.**	(b)	**94.**	(a)	**95.**	(c)	**96.**	(b)	**97.**	(b)	**98.**	(c)	**99.**	(a)	**100.**	(a)

Explanations

1. (a) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= (\sin^2\theta + \cos^2\theta)^3 = 1$

2. (b) $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1+\cot^2\theta}{\cot\theta}\right)$$

$$= \frac{\cos^2\theta\sin^2\theta\operatorname{cosec}^2\theta}{\sin\theta\cos\theta\cot\theta}$$

$$= \cos\theta\sin\theta\times\frac{1}{\sin^2\theta}\frac{\sin\theta}{\cos\theta} = 1$$

3. (b) $\left(\dfrac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}\right)^2 = \left(\dfrac{1+\sin\theta-\cos\theta}{\sin\theta-1+\cos\theta}\right)^2$

$$= \left[\frac{\sin\theta+1(1-\cos\theta)}{\sin\theta-(1-\cos\theta)}\right]^2$$

$$= \left[\frac{2\sin\theta/2\cos\theta/2+2\sin^2\theta/2}{2\sin\theta/2\cos\theta/2-2\sin^2\theta/2}\right]^2$$

$$= \left[\frac{\cos\theta/2+\sin\theta/2}{\cos\theta/2-\sin\theta/2}\right]^2 = \frac{1+\sin\theta}{1-\sin\theta}$$

4. (b) $\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}} + \sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} = \dfrac{1+\cos\alpha+1-\cos\alpha}{\sqrt{(1-\cos\alpha)(1+\cos\alpha)}}$

$$= \frac{2}{\sin\alpha} = 2\operatorname{cosec}\alpha$$

$\because \quad \pi \le \alpha < \dfrac{3\pi}{2}$

In III quadrant, cosec α is (–) ve. So, correct answer is $= -2\operatorname{cosec}\alpha$

5. (a) $\cos\theta = -\dfrac{1}{2}$ and θ is in II quadrant.

So, $\sin\theta = \dfrac{\sqrt{3}}{2}$ and $\tan\theta = -\sqrt{3}$

$$2\sin\theta + \tan\theta = 2\left(\frac{\sqrt{3}}{2}\right) + (-\sqrt{3}) = 0$$

6. (d) $\cos 24° + \cos 55° + \cos 125° + \cos 204° + \cos 300°$

$= \cos 24° + \cos 55° + \cos(180° - 55°) + \cos(180° + 24°) + \cos(360° - 60°)$

$= \cos 24° + \cos 55° - \cos 55° - \cos 24° + \cos 60°$

$= \dfrac{1}{2}$

7. (c) $x = h + p\sec\alpha \Rightarrow \dfrac{x-h}{p} = \sec\alpha$ ...(i)

$y = k + q\operatorname{cosec}\alpha \Rightarrow \dfrac{y-k}{q} = \operatorname{cosec}\alpha$ ...(ii)

From eqs. (i) and (ii)

$\dfrac{p}{x-h} = \cos\alpha, \dfrac{q}{y-k} = \sin\alpha$

$\Rightarrow \quad \sin^2\alpha + \cos^2\alpha = 1$

$$\frac{p^2}{(x-h)^2} + \frac{q^2}{(y-k)^2} = 1$$

8. (c) $\tan 20° + \tan 40° + \tan 60° + ... + \tan 180°$

$= \tan 20° + \tan 40° + ... \tan(180° - 20°)$ $\{\because \tan 180° = 0\}$

$= \tan 20° + \tan 40° + ... - \tan 40° - \tan 20° = 0$

9. (a) $\sin 10° + \sin 20° + \sin 30° + ... \sin 350° + \sin 360°$

$= \sin 10° + \sin 20° + \sin 30° + ... + \sin(360° - 30°)$

$+ \sin(360° - 20°) + \sin(360° - 10°) + \sin 360°$
$= \sin 10° + \sin 20° + \sin 30° + ... - \sin 30° - \sin 20° - \sin 10°$
$= 0$ $\{\because \sin 360° = 0\}$

10. (d) $\sin^2 5° + \sin^2 10° + \sin^2 15° + ... + \sin^2 85° + \sin^2 90°$
$= \sin^2 5° + \sin^2 10° + ... + \cos^2 10° + \cos^2 5° + 1$
$= 8 \times 1 + \sin^2 45° + 1$ $\{\because \sin(90° - \theta) = \cos\theta\}$
$= 8 + \frac{1}{2} + 1 = 9 + \frac{1}{2} = 9\frac{1}{2}$

11. (a) $45° = 75° - 30° \Rightarrow \tan 45° = \tan(75° - 30°)$
$\Rightarrow \quad 1 = \frac{\tan 75° - \tan 30°}{1 + \tan 75° \tan 30°}$
$\Rightarrow \quad 1 + \tan 75° \tan 30° = \tan 75° - \tan 30°$
or $\tan 75° - \tan 30° - \tan 75° \tan 30° = 1$

12. (b) $\sqrt{3}\ \text{cosec}\ 20° - \sec 20°$
$= 2\left(\frac{\sqrt{3}}{2}\text{cosec}\ 20° - \frac{1}{2}\sec 20°\right)$
$= 2\left(\frac{\sqrt{3}}{2\sin 20°} - \frac{1}{2\cos 20°}\right)$
$= \frac{2\left[\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right]}{\sin 20° \cos 20°}$
$= 2 \times 2\frac{[\sin 60° \cos 20° - \cos 60° \sin 20°]}{2\sin 20° \cos 20°}$
$= \frac{4[\sin(60° - 20°)]}{\sin 40°} = 4$

13. (d) $58° - 13° = 45° \Rightarrow \tan(58° - 13°) = \tan 45°$
$\Rightarrow \frac{\tan 58° - \tan 13°}{1 + \tan 58° \tan 13°} = 1$
$\Rightarrow \quad \tan 58° - \tan 13° = 1 + \tan 58° \tan 13°$
$\Rightarrow \quad \tan 58° - \tan 13° - \tan 58° \tan 13° = 1$

14. (c) $2\cos\alpha - \cos 3\alpha - \cos 5\alpha - 16\cos^3\alpha \sin^2\alpha$
$= 2\cos\alpha - [\cos 3\alpha + \cos 5\alpha] - 16\cos^3\alpha\sin^2\alpha$
$= 2\cos\alpha - [2\cos 4\alpha \cos\alpha] - 16\cos^3\alpha\sin^2\alpha$
$= 2\cos\alpha[1 - \cos 4\alpha] - 16\cos^3\alpha\sin^2\alpha$
$= 2\cos\alpha[2\sin 2\alpha] - 16\cos^3\alpha\sin^2\alpha$
$= 4\cos\alpha[\sin 2\alpha]^2 - 16\cos^3\alpha\sin^2\alpha$
$= 16\cos^3\alpha\sin^2\alpha - 16\cos^3\alpha\sin^2\alpha = 0$

15. (b) Given, $\alpha < 90°, \beta < 90°$ {acute}
$\tan\alpha = \frac{1}{7}, \sin\beta = \frac{1}{\sqrt{10}}$
$\Rightarrow \quad \tan\beta = \frac{1}{3}$
and $\tan 2\beta = \frac{2\tan\beta}{1 - \tan^2\beta} = \frac{2 \times \frac{1}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{3}} = \frac{3}{4}$
$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha \tan 2\beta}$
$\tan(\alpha + 2\beta) = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} = \frac{25}{25} = 1 \Rightarrow \alpha + 2\beta = 45°$

16. (c) $\alpha + \beta = 90° \Rightarrow \beta = 90° - \alpha$
Let $X = \cos\alpha\cos\beta = \cos\alpha\cos(90° - \alpha)$
$= \cos\alpha\ \sin\alpha = \frac{1}{2}\ \sin 2\alpha$
Now we know, $-1 \le \sin 2\alpha \le 1$
$\Rightarrow \quad = -\frac{1}{2} \le \frac{1}{2}\sin 2\alpha \le \frac{1}{2}$
Hence, min. value of $= -\frac{1}{2}$
and max. value $= \frac{1}{2}$

17. (d) $\sin x + \sin^2 x = 1 \Rightarrow \cos^2 x = \sin x$
On squaring, $\cos^4 x = \sin^2 x = 1 - \cos^2 x$
or $\cos^4 x + \cos^2 x = 1$
Again on squaring,
$\cos^8 x + 2\cos^6 x + \cos^4 x = 1$

18. (d) $\sin x + \sin^2 x = 1$
$\Rightarrow \quad \cos^2 x = \sin x$ …(i)
On squaring, $\cos^4 x = \sin^2 x = 1 - \cos^2 x$
or $\cos^4 x + \cos^2 x = 1$
On cubing
$\cos^{12} x + \cos^6 x + 3\cos^6 x(\cos^4 x + \cos^2 x) = 1$
$\Rightarrow \quad \cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x = 1$
$\Rightarrow \quad \cos^{12} x + \cos^{10} x + 3\cos^8 x + \cos^6 x + 2\cos^4 x + \cos^2 x - 2$
$= 1 + 2\cos^4 x + \cos^2 x - 2$
$= 2\cos^4 x - (1 - \cos^2 x) = 2\cos^4 x - \sin^2 x$
$= 2\sin^2 x - \sin^2 x$ {From eq. (i)}
$= \sin^2 x$

19. (c) $32\sin\frac{A}{2}\sin\frac{5A}{2}$
$= 16\left\{\cos\left(\frac{A}{2} - \frac{5A}{2}\right) - \cos\left(\frac{A}{2} + \frac{5A}{2}\right)\right\}$
$= 16\{\cos 2A - \cos 3A\} = 16\left\{\frac{1}{8} + \frac{9}{16}\right\}$
$\because \quad \cos A = \frac{3}{4}$
$\Rightarrow \cos 2A = 2\left(\frac{9}{16}\right) - 1 = \frac{1}{8}$
and $\cos 3A = 4\left(\frac{27}{64}\right) - 3\left(\frac{3}{4}\right) = -\frac{9}{16}$
$= 16\left\{\frac{11}{16}\right\} = 11$

20. (b) $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{1-\cos 8\theta}{1-\cos 4\theta}\cdot\dfrac{\cos 4\theta}{\cos 8\theta}$

$= \dfrac{2\sin^2 4\theta\cos 4\theta}{2\sin^2 2\theta\cos 8\theta} = \dfrac{(\sin 4\theta)(2\sin 4\theta\cos 4\theta)}{(2\sin^2 2\theta)(\cos 8\theta)}$

$= \dfrac{2\sin 2\theta\cos 2(\theta)}{2\sin^2 2\theta}\cdot\left(\dfrac{\sin 8\theta}{\cos 8\theta}\right)$

$= \cot 2\theta\cdot\tan 8\theta = \dfrac{\tan 8\theta}{\tan 2\theta}$

21. (d) $\cot(\alpha+\beta) = 0 = \cot 90°$

$\Rightarrow \alpha + \beta = 90°$ or $\alpha = 90° - \beta$

Then $\sin(\alpha + 2\beta) = \sin(90° - \beta + 2\beta)$

$= \sin(90° + \beta) = \cos\beta$

22. (b) $A = \sin 26°$

$B = \cos 26° = \cos(90° - 64°) = \sin 64°$

$\because$ Sine is an increasing function.

So, $\sin 64° > \sin 26°$ or $\cos 26° > \sin 26°$

23. (a) $\tan A + \tan B + \tan C = \tan A \times \tan B \times \tan C$

$\Rightarrow \tan A + \tan B = -\tan C(1 - \tan A\tan B)$

$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A\tan B} = \tan(180 - C)$

$\Rightarrow \tan(A+B) = \tan(180° - C)$

$\Rightarrow A + B + C = 180°$

24. (c) $\cos^4\dfrac{\pi}{8} + \cos^4\dfrac{3\pi}{8} + \cos^4\dfrac{5\pi}{8} + \cos^4\dfrac{7\pi}{8}$

$= \cos^4\dfrac{\pi}{8} + \cos^4\dfrac{3\pi}{8} + \cos^4\left(\pi - \dfrac{3\pi}{8}\right) + \cos^4\left(\pi - \dfrac{\pi}{8}\right)$

$= 2\left(\cos^4\dfrac{\pi}{8} + \cos^4\dfrac{3\pi}{8}\right) = 2\left\{\cos^4\dfrac{\pi}{8} + \sin^4\dfrac{\pi}{8}\right\}$

$= 2\left\{\left(\cos^2\dfrac{\pi}{8} + \sin^2\dfrac{\pi}{8}\right)^2 - 2\sin^2\dfrac{\pi}{8}\cos^2\dfrac{\pi}{8}\right\}$

$= 2\left\{1 - \dfrac{1}{2}\sin^2\dfrac{\pi}{4}\right\} = 2\left\{1 - \dfrac{1}{4}\right\} = \dfrac{3}{2}$

25. (c) $\sin A + \sin B = a$ and $\cos A + \cos B = b$

$a^2 = \sin^2 A + \sin^2 B + 2\sin A\sin B$

$b^2 = \cos^2 A + \cos^2 B + 2\cos A\cos B$

Now, $b^2 - a^2 = \cos 2A + \cos 2B + 2\cos(A+B)$

or $b^2 - a^2 = 2\cos(A+B)\{\cos(A-B) + 1\}$

and $b^2 + a^2 = 1 + 1 + 2\cos(A-B)$

or $b^2 + a^2 = 2\{\cos(A-B) + 1\}$

$\Rightarrow \dfrac{b^2 - a^2}{b^2 + a^2} = \cos(A+B)$

26. (b) $\dfrac{\cos 2B}{1} = \dfrac{\cos(A+C)}{\cos(A-C)}$

Applying Componendo and Dividendo theorem

$\dfrac{1+\cos 2B}{1-\cos 2B} = \dfrac{\cos(A-C)+\cos(A+C)}{\cos(A-C)-\cos(A+C)}$

or $\dfrac{2\cos^2 B}{2\sin^2 B} = \dfrac{2\cos A\cos C}{2\sin A\sin C}$ or $\tan^2 B = \tan A\tan C$

$\Rightarrow \tan A$, $\tan B$ and $\tan C$ are in GP.

27. (c) Given, $\tan^2\theta = 2\tan^2\phi + 1$ …(i)

Then, $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \cos 2\theta = \dfrac{-2\tan^2\phi}{2(1+\tan^2\phi)}$ [from Eq. (i)]

$\Rightarrow \cos 2\theta = \dfrac{-\sin^2\phi}{\sin^2\phi + \cos^2\phi}$

or $\cos 2\theta + \sin^2\phi = 0$

28. (b) $\cos\dfrac{\pi}{9}\cos\dfrac{2\pi}{9}\cos\dfrac{3\pi}{9}\cos\dfrac{4\pi}{9}$

$= \left\{\cos\dfrac{\pi}{9}\cos\dfrac{2\pi}{9}\cos\dfrac{4\pi}{9}\right\}\cos\dfrac{3\pi}{9}$

$= \dfrac{1}{2}\left\{\dfrac{\sin\left(2^3.\dfrac{\pi}{9}\right)}{2^3\sin\left(\dfrac{\pi}{9}\right)}\right\} = \dfrac{1}{16}\dfrac{\sin\left(\dfrac{8\pi}{9}\right)}{\sin\left(\dfrac{\pi}{9}\right)} = \dfrac{1}{16}$

29. (b) $\cos\dfrac{\pi}{15}\cos\dfrac{2\pi}{15}\cos\dfrac{3\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{5\pi}{15}\cos\dfrac{6\pi}{15}\cos\dfrac{7\pi}{15}$

$= -\cos\dfrac{\pi}{3}\left\{\cos\dfrac{\pi}{15}\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{8\pi}{15}\right\}$

$\left\{\cos\dfrac{3\pi}{15}\cos\dfrac{6\pi}{15}\right\}$

$\left\{\because \cos\dfrac{7\pi}{15} = \cos\left(\pi - \dfrac{8\pi}{15}\right) = -\cos\dfrac{8\pi}{15}\right\}$

$= -\dfrac{1}{2}\dfrac{\sin\left(2^4.\dfrac{\pi}{15}\right)}{2^4\sin\dfrac{\pi}{15}}\cdot\dfrac{\sin\left(2^2\dfrac{3\pi}{15}\right)}{2^2\sin^2\dfrac{3\pi}{15}}$

$= \left(-\dfrac{1}{128}\right)\dfrac{\sin\left(\dfrac{16\pi}{15}\right)}{\sin\left(\dfrac{\pi}{15}\right)}\cdot\dfrac{\sin\left(\dfrac{12\pi}{15}\right)}{\sin\left(\dfrac{3\pi}{15}\right)} = \dfrac{1}{128} = \dfrac{1}{2^7}$

30. (c) $A = \tan 6°\tan 42°$ and $B = \cot 66°\cot 78°$

$\dfrac{A}{B} = \dfrac{\sin 6^o\sin 42^o\sin 66^o\sin 78^o}{\cos 6^o\cos 42^o\cos 66^o\cos 78^o}$

$= \dfrac{(2\sin 6^o\sin 66^o)(2\sin 42^o\sin 78^o)}{(2\cos 6^o\cos 66^o)(2\cos 42^o\cos 78^o)}$

$= \dfrac{(\cos 60^o - \cos 72^o)(\cos 36^o - \cos 120^o)}{(\cos 72^o + \cos 60^o)(\cos 120^o + \cos 36^o)}$

$$= \frac{\left\{\frac{1}{2}-\frac{\sqrt{5}-1}{4}\right\}\left\{\frac{\sqrt{5}+1}{4}+\frac{1}{2}\right\}}{\left\{\frac{\sqrt{5}-1}{4}+\frac{1}{2}\right\}\left\{\frac{-1}{2}+\frac{\sqrt{5}+1}{4}\right\}}$$

$$= \frac{(3-\sqrt{5})(3+\sqrt{5})}{(\sqrt{5}+1)(\sqrt{5}-1)} = 1 \Rightarrow A = B$$

31. (a) $\sin(\alpha+\beta) = 1$ and $\sin(\alpha - \beta) = \frac{1}{2}$

$\Rightarrow \alpha+\beta = 90°$ and $\alpha-\beta = 30°$

$\Rightarrow \alpha = 60°$ and $\beta = 30°$

So, $\tan(\alpha+2\beta)\tan(2\alpha+\beta) = \tan 120° \tan 150°$

$= (-\tan 60°)(-\tan 30°) = 1$

32. (c) $m = \sin^6 x + \cos^6 x$

$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$

$\Rightarrow m = 1 - 3\sin^2 x\cos^2 x = 1 - \frac{3}{4}\sin^2 2x$

$\because 0 \le \sin^2 2x \le 1$

$\Rightarrow \frac{1}{4} \le 1 - \frac{3}{4}\sin^2 \le 1 \Rightarrow \frac{1}{4} \le m \le 1$

33. (c) $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

$= 3(1-2\sin x\cos x)^2 + 6(1+2\sin x\cos x) + 4(1-3\sin^2 x\cos^2 x)$

$= 3\{1+4\sin^2 x\cos^2 x - 4\sin x\cos x\} + 6\{1 + 2\sin x\cos x\} + 4\{1-3\sin^2 x\cos^2 x\}$

$= 13$

34. (c) $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$

$= \sin\theta(\sin\theta + 3\sin\theta - 4\sin^3\theta)$

$= 4\sin^2\theta(1-\sin^2\theta)$

$f(\theta) = \sin^2 2\theta$

So, $f(\theta) \ge 0\ \forall$ real value of θ

35. (c) $p = \sin A + \cos A$ and $q = \sin^3 A + \cos^3 A$

Then, $p^3 = \sin^3 A + \cos^3 A + 3\sin A\cos A(\sin A + \cos A)$

or $p^3 - q = 3p\sin A\cos A$ …(i)

Similarly, $p^2 = \sin^2 A + \cos^2 A + 2\sin A\cos A$

or $p^2 - 1 = 2\sin A\cos A$ …(ii)

From eq. (i) ÷ eq. (ii),

$$\frac{p^3-q}{p^2-1} = \frac{3p}{2}$$

$\Rightarrow 2p^3 - 2q = 3p^3 - 3p$ or $p^3 - 3p + 2q = 0$

36. (c) $\sin(x+3\alpha) = 3\sin(\alpha - x)$

$\sin x\cos 3\alpha + \cos x\sin 3\alpha = 3\sin\alpha\cos x - 3\cos\alpha\sin x$

$\Rightarrow \sin x\{\cos 3\alpha + 3\cos\alpha\} = \cos x\{3\sin\alpha - \sin 3\alpha\}$

$\Rightarrow \sin x\{4\cos^3\alpha\} = \cos x\{4\sin^3\alpha\}$

$\Rightarrow \tan x = \tan^3\alpha$

37. (d) $\sin^3 10° + \sin^3 50° - \sin^3 70°$

$\{\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta\}$

$= \frac{1}{4}[3\sin 10° - \sin 30° + 3\sin 50° - \sin 150° - 3\sin 70° + \sin 210°]$

$$= \frac{1}{4}\left[3\{\sin 10° + \sin 50° - \sin 70°\} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2}\right]$$

$$= \frac{1}{4}\left[3\{\sin 10° - 2\cos 60°\sin 10°\} - \frac{3}{2}\right]$$

$$= \frac{1}{4}\left[3\{\sin 10° - \sin 10°\} - \frac{3}{2}\right] = -\frac{3}{8}$$

38. (a) $A = \sin 45° + \cos 45°$

$$A = \sqrt{2}\left\{\frac{1}{\sqrt{2}}\sin 45° + \frac{1}{\sqrt{2}}\cos 45°\right\} = \sqrt{2}\sin 90°$$

$B = \sin 44° + \cos 44°$

$$= \sqrt{2}\left\{\frac{1}{\sqrt{2}}\sin 44° + \frac{1}{\sqrt{2}}\cos 44°\right\} = \sqrt{2}\sin 89°$$

$\because \sin 90° > \sin 89° \Rightarrow A > B$

39. (a) $\sin x = +\frac{1}{2} = \sin 30° = \sin 150°$

$\because$ x lies in II quadrant

So, $x = 150° \Rightarrow \frac{x}{2} = 75°$

$\therefore \cos 75° = \cos(90° - 15°) = \sin 15°$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{(\sqrt{2-\sqrt{3}})\sqrt{2}}{2\sqrt{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

40. (c) $\frac{\sin 50° - \sin 40°}{\cos 50° - \cos 40°} \Rightarrow \frac{\sin 50° - \sin 40°}{\sin 40° - \sin 50°} = -1$

41. (a) $\cos 10° - \sin 10°$

$= \cos(90° - 80°) - \sin 10° = \sin 80° - \sin 10°$

$\because$ sin is an increasing function

So, $\sin 80° > \sin 10°$

$\Rightarrow \sin 80° - \sin 10° > 0$

42. (b) $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= 4\tan\theta\sin\theta$

$mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= \tan^2\theta - \sin^2\theta$ ...(i)

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$$

$= \sin^4\theta\cos^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta}\sin^2\theta = \tan^2\theta\sin^2\theta$$

$\Rightarrow \tan\theta\sin\theta = \sqrt{mn}$

Put in, we get (i)

$m^2 - n^2 = 4\sqrt{mn}$

43. (b) $\sin\alpha + \text{cosec}\,\alpha = 2$

On squaring,

$\sin^2\alpha + \text{cosec}^2\alpha + 2\sin\alpha\,\text{cosec}\,\alpha = 4$

$\Rightarrow \sin^2\alpha + \text{cosec}^2\alpha = 4 - 2 = 2$

44. (b) $\dfrac{\sin A + \sin 3A}{\cos A + \cos 3A} = \dfrac{2\sin 2A\cos A}{2\cos 2A\cos A} = \tan 2A$

45. (b) $\tan\dfrac{\pi}{20}\tan\dfrac{3\pi}{20}\tan\dfrac{5\pi}{20}\tan\dfrac{7\pi}{20}\tan\dfrac{9\pi}{20}$

$= \tan\dfrac{\pi}{20}\tan\dfrac{3\pi}{20}\tan\dfrac{\pi}{4}\tan\left(\dfrac{\pi}{2}-\dfrac{3\pi}{20}\right)\tan\left(\dfrac{\pi}{2}-\dfrac{\pi}{20}\right)$

$= \tan\dfrac{\pi}{20}\tan\dfrac{3\pi}{20}\cot\dfrac{3\pi}{20}\cot\dfrac{\pi}{20} = 1$

46. (b) $\because\ \theta + (2n-1)\theta = 2n\theta = \dfrac{\pi}{2}$ $\{\because\ \theta = \dfrac{\pi}{4}n\}$

and $2\theta + (2n-2)\theta = 2n\theta = \dfrac{\pi}{2}$

So, $\tan\theta\tan 2\theta \ldots\ldots \tan(2n-2)\theta\tan(2n-1)\theta = 1$

47. (b) $\tan 20° + 2\tan 50° - \tan 70°$

$= 2\tan 50° - \{\tan 70° - \tan 20°\}$

$= 2\tan 50° - \{\cot 20° - \tan 20°\}$

$= 2\tan 50° - 2\tan 50° = 0$

48. (d) $f(\theta) = 4(\sin^2\theta + \cos^4\theta)$

$f(\theta) = 4(\cos^4\theta - \cos^2\theta + 1)$

$= 4\left\{\left(\cos^2\theta - \dfrac{1}{2}\right)^2 + 1 - \dfrac{1}{4}\right\}$

$= 4\left\{\left(\cos^2\theta - \dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2\right\}$

$f(\theta) = 4\left(\cos^2\theta - \dfrac{1}{2}\right)^2 + 3$

So, maximum value of $f(\theta) = 1 + 3 = 4$

49. (d) From above question minimum value of $f(\theta) = 0 + 3 = 3$

50. (b) From above question $f(\theta) = 2$

$\Rightarrow 4\left(\cos^2\theta - \dfrac{1}{2}\right)^2 + 3 = 2$

$\left(\cos^2\theta - \dfrac{1}{2}\right)^2 = -\dfrac{1}{2}$ is not possible.

So, $f(\theta) = 2$ has no solution.

$f(\theta) = \dfrac{7}{2}$.

$\Rightarrow 4\left(\cos^2\theta - \dfrac{1}{2}\right)^2 + 3 = \dfrac{7}{2}$

$\Rightarrow \left(\cos^2\theta - \dfrac{1}{2}\right)^2 = \dfrac{1}{8}$

$\Rightarrow \cos^2\theta - \dfrac{1}{2} = \dfrac{1}{2\sqrt{2}} \Rightarrow \cos^2\theta = \dfrac{1+\sqrt{2}}{2\sqrt{2}}$

Hence, there is a solution. So, only 2 is true.

51. (c) $\dfrac{A}{B} = \dfrac{(\cos 12° - \cos 36°)(\sin 96° + \sin 24°)}{(\sin 60° - \sin 12°)(\cos 48° - \cos 72°)}$

$= \dfrac{2\sin 24°\sin 12° \times 2\sin 60°\cos 36°}{2\cos 36°\sin 24° \times 2\sin 60°\sin 12°} = 1$

52. (d) $\tan\alpha$ and $\tan\beta$ are the roots of $x^2 + bx + c = 0$, $b \neq 0$

$\Rightarrow \tan\alpha + \tan\beta = -b$ and $\tan\alpha\tan\beta = c$

So, $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{-b}{1-c} = b(c-1)^{-1}$

53. (b) $\sin(\alpha+\beta)\sec\alpha\sec\beta$

$= \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}$

$= \tan\alpha + \tan\beta = -b$

54. (d) $\cos A + \cos B + \cos C = \sqrt{3}\sin\dfrac{\pi}{3} = \dfrac{3}{2}$

$\Rightarrow A = B = C = 60°$

$\Rightarrow \sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2} = \sin 30°\sin 30°\sin 30°$

$= \dfrac{1}{8}$.

55. (d) $\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{B+C}{2}\right)\cos\left(\dfrac{C+A}{2}\right)$

$= \cos 60°\cos 60°\cos 60°$

{from above question $A = B = C = 60°$}

$= \dfrac{1}{8}$

56. (b) 1. $A = B = C = 60°$

$3\tan(A+B)\tan C = 3\tan 120°\tan 60°$

$= -3\tan 60°\tan 60° = -3 \times 3 = -9$

2. $A = 78°, B = 66°$

$\Rightarrow C = 180° - (78° + 66°)$

$\Rightarrow C = 36°$

$\tan\left(\dfrac{A}{2} + C\right) = \tan(39° + 36°) = \tan 75°$

$\tan A = \tan 78°$

$\Rightarrow \tan A > \tan\left(\dfrac{A}{2} + C\right)$

3. In ΔABC, $A + B + C = 180°$

$\Rightarrow \dfrac{A+B}{2} = 90 - \dfrac{C}{2}$

$\Rightarrow \tan\left(\dfrac{A+B}{2}\right) = \cot\dfrac{C}{2}$

$\Rightarrow \tan\left(\dfrac{A+B}{2}\right)\sin\dfrac{C}{2} = \cot\dfrac{C}{2} \times \sin\dfrac{C}{2} = \cos\dfrac{C}{2}$

Hence, only statement 2 is correct.

57. (b) $k \sin x + \cos 2x = 2k - 7$

$\Rightarrow k \sin x + 1 - 2 \sin^2 x = 2k - 7$

$\Rightarrow 2 \sin^2 x - k \sin x + 2k - 8 = 0$

$$\Rightarrow \sin x = \frac{k \pm \sqrt{k^2 - 16k + 64}}{4}$$

$$\Rightarrow \sin x = \frac{k \pm (k-8)}{4}$$

$$\Rightarrow \sin x = \frac{2k-8}{4} \text{ and } \sin x = 2 \text{ \{not possible\}}$$

$-1 \le \sin x \le 1$

$$\Rightarrow -1 \le \frac{2k-8}{4} \le 1 \Rightarrow -4 \le 2k - 8 \le 4$$

$\Rightarrow 2 \le k \le 6$

So minimum value of $k = 2$

58. (d) From the solution in Q. 57.

Maximum value of $k = 6$

59. (d) $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$

$= \{\tan 9° + \cot 9°\} - \{\tan 27° + \cos 27°\}$

$$= \left\{\frac{\sin^2 9° + \cos^2 9°}{\sin 9° + \cos 9°}\right\} - \left\{\frac{\sin^2 27° + \cos^2 27°}{\sin 27° + \cos 27°}\right\}$$

$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°} = \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1}$$

$$= 8\left\{\frac{\sqrt{5}+1-\sqrt{5}+1}{5-1}\right\} = 4$$

60. (a) $a \cos \theta + b \sin \theta = c$ ...(i)

Let $a \sin \theta - b \cos \theta = x$ ...(ii)

From eqs. $(i)^2 + (ii)^2$

$a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = c^2 + x^2$

$\Rightarrow x^2 = a^2 + b^2 - c^2$

or $(a \sin \theta - b \cos \theta)^2 = a^2 + b^2 - c^2$

61. (b) $$\frac{1 - \tan 2 \cot 62°}{\tan 152° - \cot 88°} = k\sqrt{3}$$

$$\Rightarrow \frac{1 - \frac{\tan 2°}{\tan 62°}}{\tan(90° + 62°) - \cot(90° - 2°)} = k\sqrt{3}$$

$$\Rightarrow \frac{\frac{(\tan 62° - \tan 2°)}{\tan 60°}}{-\cot 62° - \tan 2°} = k\sqrt{3}$$

$$\Rightarrow \frac{\tan 62° - \tan 2°}{-\tan 62°\left[\frac{1 + \tan 2° \tan 62°}{\tan 62°}\right]} = k\sqrt{3}$$

$\Rightarrow -\tan (62° - 2°) = k\sqrt{3}$

$\Rightarrow -\tan 60° = k\sqrt{3}$

$\Rightarrow -\sqrt{3} = k\sqrt{3} \Rightarrow k = -1$

62. (a) $$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

$$= \left(2\cos^2\frac{\pi}{16}\right)\left(2\cos^2\frac{3\pi}{16}\right)\left(2\cos^2\frac{5\pi}{16}\right)\left(2\cos^2\frac{7\pi}{8}\right)$$

$$= 16\left[\cos\frac{\pi}{16}\cos\frac{3\pi}{16}\cos\frac{5\pi}{16}\cos\frac{7\pi}{16}\right]^2$$

$$\left[\begin{array}{l} \therefore \cos\frac{7\pi}{16} = \cos\left(\frac{\pi}{2} - \frac{\pi}{16}\right) = \sin\frac{\pi}{16} \\ \text{and} \cos\frac{5\pi}{16} = \cos\left(\frac{\pi}{2} - \frac{3\pi}{16}\right) = \sin\frac{3\pi}{16} \end{array}\right]$$

$$= 16\left[\cos\frac{\pi}{16}\sin\frac{\pi}{16}\cos\frac{3\pi}{16}\sin\frac{3\pi}{16}\right]^2$$

$$= \left[\left(2\cos\frac{\pi}{16}\sin\frac{\pi}{16}\right)\left(2\cos\frac{3\pi}{16}\sin\frac{3\pi}{16}\right)\right]^2$$

$$= \frac{1}{4}\times\left[4\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}\right] = \frac{1}{4}\left[2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right]^2$$

$$= \frac{1}{4}\left[\cos\left(\frac{\pi}{8} - \frac{3\pi}{8}\right) - \cos\left(\frac{\pi}{8} + \frac{3\pi}{8}\right)\right]^2$$

$$= \frac{1}{4}\left[\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right]^2 = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

63. (c) $\sin A + 2 \sin 2A + \sin 3A$

$= (\sin A + \sin 3A) + 2 \sin 2A$

$= 2 \sin 2A \cos A + 2 \sin 2A$

$= 2 \sin 2A \{\cos A + 1\}$

$$= 4 \sin 2A \cos^2 \frac{A}{2}$$

$$= 8 \sin A \cos A \cos^2 \frac{A}{2}$$

So, only 1 and 3 are correct.

64. (a) $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 + \sin \theta_4 = 4$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \theta_4 = 90°$

$\Rightarrow \cos \theta_1 + \cos \theta_2 + \cos \theta_3 + \cos \theta_4 = 0$

65. (d) $\sin A = \frac{3}{5}$, $450° < A < 540°$

$\Rightarrow A$ is in II quadrant.

So, $\cos A = -\frac{4}{5}$

$$\Rightarrow 2\cos^2\frac{A}{2} - 1 = -\frac{4}{5} \Rightarrow 2\cos^2\left(\frac{A}{2}\right) = \frac{1}{5}$$

$$\Rightarrow \cos\frac{A}{2} = -\frac{1}{\sqrt{10}}\left\{\therefore 225° < \frac{A}{2} < 270°\right\}$$

66. (d) $$\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°}$$

$$= \frac{2\times\left\{\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right\}\times 2}{2\sin 10° \cos 10°}$$

$$= \frac{4\times\{\sin 30^o\cos 10^o - \cos 30^o \sin 10^o\}}{\sin 2\times 10^o}$$

$$= \frac{4\sin(30^o-10^o)}{\sin 20^o} = 4$$

67. (b) $\sin\theta = 3\sin(\theta+2\alpha)$

$$\frac{\sin\theta}{\sin(\theta+2\alpha)} = \frac{3}{1}$$

$$\Rightarrow \frac{\sin\theta+\sin(\theta+2\alpha)}{\sin\theta-\sin(\theta+2\alpha)} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{2\sin(\theta+\alpha)\cos\alpha}{-2\cos(\theta+\alpha)\sin\alpha} = \frac{4}{2}$$

$$\Rightarrow \frac{\tan(\alpha+\theta)}{\tan\theta} = -2$$

$\Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$

68. (a) $\tan 2\alpha = \tan\{(\alpha+\beta)+(\alpha-\beta)\}$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{2+1}{1-2} = -3$$

69. (a) $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}$

$$= \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)} = \tan\left(\frac{\alpha+\beta}{2}\right)$$

70. (b) In ΔABC, $A + B + C = 180°$

1. $\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{180^o - A}{2}\right)$

   $= \sin\left(90^o - \frac{A}{2}\right) = \cos\frac{A}{2}$

2. $\tan\left(\frac{B+C}{2}\right) = \tan\left(\frac{180^o-A}{2}\right) = \tan\left(90^o - \frac{A}{2}\right)$

   $= \cot\frac{A}{2}$

3. $\sin(B+C) = \sin(180° - A) = \sin A$
4. $\tan(B+C) = \tan(180° - A) = -\tan A$

Hence, only 1 and 2 are correct.

71. (b) $\sec\theta - \text{cosec}\,\theta = \dfrac{4}{3}$

$\Rightarrow \sin\theta - \cos\theta = \dfrac{4}{3}\sin\theta\cos\theta$

$\Rightarrow 1 - \sin 2\theta = \dfrac{4}{9}\sin^2 2\theta$

$\Rightarrow 4\sin^2 2\theta + 9\sin 2\theta - 9 = 0$

$\Rightarrow (4\sin 2\theta - 3)(\sin 2\theta + 3) = 0$

$\sin 2\theta = \dfrac{3}{4}$ and $\sin 2\theta = -3$ [not possible]

Hence, $(\sin\theta - \cos\theta)^2 = 1 - \sin 2\theta = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

$\Rightarrow \sin\theta - \cos\theta = \pm\dfrac{1}{2}$

72. (c) Let $A = 3\sin\theta + 4\cos\theta$

$\because\ \sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

$\Rightarrow -\sqrt{3^2+4^2} \le 3\sin\theta + 4\cos\theta \le \sqrt{3^2+4^2}$

$-5 \le 3\sin\theta + 4\cos\theta \le 5$

So, Minimum value = – 5

73. (a) $\dfrac{\sin^2 A+\sin^2 B+\sin^2 C}{\cos^2 A+\cos^2 B+\cos^2 C} = 2$

$$\Rightarrow \frac{3-(\cos^2 A+\cos^2 B+\cos^2 C)}{\cos^2 A+\cos^2 B+\cos^2 C} = 2$$

$\Rightarrow 3 - (\cos^2 A + \cos^2 B + \cos^2 C)$

$= 2(\cos^2 A + \cos^2 B + \cos^2 C)$

$\Rightarrow \cos^2 A + \cos^2 B + \cos^2 C = 1$

$\Rightarrow \Delta ABC$ is a right angled triangle.

74. (c) $\because\ \tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$

$\Rightarrow \tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a}$ and $\tan\frac{P}{2} + \tan\frac{Q}{2} = \frac{c}{a}$

$\therefore\quad \angle R = 90° \Rightarrow \angle P + \angle Q = 90°$

or $\tan\left(\frac{P}{2}+\frac{Q}{2}\right) = \tan 45^o$

$$\Rightarrow \frac{\tan\frac{P}{2}+\tan\frac{Q}{2}}{1-\tan\frac{P}{2}\tan\frac{Q}{2}} = 1 \Rightarrow \frac{-\frac{b}{a}}{1-\frac{c}{a}} = 1$$

$$\Rightarrow \frac{b}{c-a} = 1 \Rightarrow c = a+b$$

75. (d) $\dfrac{\tan A}{\tan B} = \dfrac{p}{q} \Rightarrow \dfrac{\sin A\cos B}{\sin B\cos A} = \dfrac{p}{q}$

$$\Rightarrow \frac{\sin A\cos B+\cos A\sin B}{\sin A\cos B-\cos A\sin B} = \frac{p+q}{p-q}$$

$$\Rightarrow \frac{\sin(A+B)}{\sin(A-B)} = \frac{p+q}{p-q}$$

$$\Rightarrow \frac{\sin\alpha}{\sin x} = \frac{p+q}{p-q}$$

$\{\because A + B = \alpha$ and $A - B = x\}$

$$\Rightarrow \sin x = \left(\frac{p-q}{p+q}\right)\sin\alpha$$

76. (c) $\sqrt{1+\sin A} = -\left(\sin\frac{A}{2}+\cos\frac{A}{2}\right)$

$$\text{RHS} = -\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\frac{A}{2}+\frac{1}{\sqrt{2}}\cos\frac{A}{2}\right)$$

$= -\sqrt{2}\sin\left(\frac{A}{2}+\frac{\pi}{4}\right)$

RHS will be positive, if $\sin\left(\frac{A}{2}+\frac{\pi}{4}\right)$ in negative

$\therefore\ \pi < \frac{A}{2}+\frac{\pi}{4} < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{A}{2} < \frac{7\pi}{4}$

$\Rightarrow \frac{3\pi}{2} < A < \frac{7\pi}{2}$

77. (a) $xy = \sin 70° \sin 50° \cos 60° \cos 80°$
$= \cos 20° \cos 40° \cos 60° \cos 80°$
$= \cos 20° \cos (60° - 20°) \cos (60° + 20°) \times \frac{1}{2}$
$= \frac{1}{4} \cos (3 \times 20°) \times \frac{1}{2} = \frac{1}{16}$

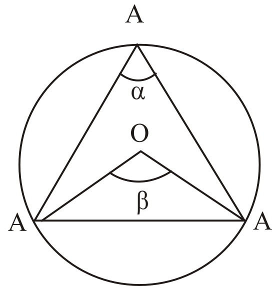
78. (c) Given, $\sin x = \frac{1}{\sqrt{5}}$ and $\sin y = \frac{1}{\sqrt{10}}$
$\Rightarrow \tan x = \frac{1}{2}$ and $y = \frac{1}{3}$
Let $\alpha = x + y$
$\Rightarrow \tan\alpha = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}}$
$\Rightarrow \tan\alpha = 1 = \tan\frac{\pi}{4}$
$\Rightarrow \alpha = \frac{\pi}{4}$

79. (c) $\frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x} = \frac{2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$
$= \frac{2\cos 4x \sin x}{2\cos 4x \cos x} = \tan x$

80. (c) $\sin 105° + \cos 105°$
$= \sin (90° + 15°) + \cos (90° + 15°)$
$= \cos 15° - \sin 15° = \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}$
$= \frac{2}{2\sqrt{2}} = \frac{1}{2}$

81. (b) Sum of angles in a triangle is 180°
So, $x + (x - y) + (x + y) = 180°$
$\Rightarrow 3x = 180°$
$\Rightarrow x = 60° = \frac{\pi}{3}$

82. (a)



Angle formed by an arc at the centre of a circle is double of the angle formed by the same arc on the circumference.
Hence, $\beta = 2\alpha$
So, $\cos\beta = \cos 2\alpha$
$\Rightarrow \cos\beta = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$

83. (a) $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$
Applying Componendo-Dividendo theorem.
$\frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$
$\Rightarrow \frac{2\sin x\cos y}{2\cos x\sin y} = \frac{2a}{2b} \Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$

84. (b) Given, $\sin\alpha + \sin\beta = 0$ and $\cos\alpha + \cos\beta = 0$
So, $(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2 = 0$
$\Rightarrow 2 + 2(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = 0$
$\Rightarrow \cos(\alpha - \beta) = -1 = \cos\pi$
$\Rightarrow \alpha - \beta = \pi \Rightarrow \alpha = \pi + \beta$
$\Rightarrow$ A must be an odd multiple of π.

85. (c) We know, $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$
Given, only one value of $\cos\frac{A}{2}$ is possible.
$\Rightarrow \cos\frac{A}{2} = 0 \Rightarrow 1 + \cos A = 0$
$\Rightarrow \cos A = -1 \Rightarrow A = (2n - 1)\pi$
$\Rightarrow$ A must be an odd multiple of π

86. (b) Given, $\cos\alpha + \cos\beta + \cos\gamma = 0$
where, $0 < \alpha \le \frac{\pi}{2}, 0 \le \beta, \le \frac{\pi}{2}$ and $0 < \gamma \le \frac{\pi}{2}$
So, $\alpha = \beta = \gamma = \frac{\pi}{2}$
Hence, $\sin\alpha \sin\beta \sin\gamma = 3 \sin\frac{\pi}{2} = 3$

87. (a) Let $A = \sin\left(x+\frac{\pi}{5}\right) + \cos\left(x+\frac{\pi}{5}\right)$
$A = \sqrt{2}\left\{\frac{1}{\sqrt{2}}\sin\left(x+\frac{\pi}{5}\right) + \frac{1}{\sqrt{2}}\cos\left(x+\frac{\pi}{5}\right)\right\}$
$= \sqrt{2}\left\{\sin\left(x+\frac{\pi}{5}+\frac{\pi}{4}\right)\right\}$

$A = \sqrt{2}\sin\left(x + \frac{9\pi}{20}\right)$

A will be maximum, if $x + \frac{9\pi}{20} = \frac{\pi}{2}$

$\therefore \quad x = \frac{\pi}{2} - \frac{9\pi}{20} = \frac{\pi}{20}$

88. (c) Let A = 16 sin θ – 12 $\sin^2$ θ

$= -12\left\{\sin^2\theta - \frac{4}{3}\sin\theta\right\}$

$= -12\left\{\sin^2\theta - 2(\sin\theta)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2\right\}$

$= -12\left\{\left(\sin\theta - \frac{2}{3}\right)^2 - \frac{4}{9}\right\}$

$= 12\left\{\frac{4}{9} - \left(\frac{2}{3} - \sin\theta\right)^2\right\}$

A will be maximum, when $\left(\frac{2}{3} - \sin\theta\right)^2$ is maximum *i.e.*, 0

So, maximum value of $A = 12 \times \frac{4}{9} = \frac{16}{3}$

89. (d) Given $A + B + C = 180°$

$\Rightarrow B + C = 180° - A$

So, sin 2A – sin 2B – sin 2C

= sin 2A – {sin 2B + sin 2C}

= sin 2A – 2 sin (B + C) cos (B – C)

= sin 2A – 2 sin (180° – A) cos (B – C) {From eq. (i)}

= 2 sin A cos A – 2 sin A cos (B – C)

= 2 sin A {cos (180° – (B + C)) – cos (B – C)}

= – 2 sin A {cos (B + C) + cos (B – C)}

= – 4 sin A cos B cos C

90. (a) By Hit and Trial Method

A = 300° satisfies the given condition

91. (b) Given, cosec x + cot x = $\sqrt{3}$ ...(i)

$\Rightarrow$ cosec x – cot x = $\frac{1}{\sqrt{3}}$ ...(ii)

{∵ $\text{cosec}^2 x = \cot^2 x = 1$}

On adding eq. (i) and (ii),

$2 \text{ cosec } x = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow \text{cosec } x = \frac{2}{\sqrt{3}} = \text{cosec}\frac{\pi}{3} \text{ or cosec}\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow x = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$

92. (c) $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

93. (b) $(2\cos\theta + 1)^{10}(2\cos 2\theta - 1)^{10}(2\cos\theta - 1)^{10}(2\cos 4\theta - 1)^{10}$

$= (4\cos^2\theta - 1)^{10}(2\cos 2\theta - 1)^{10}(2\cos 4\theta - 1)^{10}$

$= (2\cos 2\theta + 1)^{10}(2\cos 2\theta - 1)^{10}(2\cos 4\theta - 1)^{10}$

$= (4\cos^2 2\theta - 1)^{10}(2\cos 4\theta - 1)^{10}$

$= (2\cos 4\theta + 1)^{10}(2\cos 4\theta - 1)^{10}$ {∵ $\cos 2\theta = 2\cos^2\theta - 1$}

$= (4\cos^2 4\theta - 1)^{10} = (2\cos 8\theta + 1)^{10}$

$= (2\cos\pi + 1)^{10}$ {∵ $\theta = \frac{\pi}{8}$ given}

= 1

94. (a) Roots of the eq. $4x^2 - 3 = 0$ are $x = \frac{\sqrt{3}}{2}$ and $x = -\frac{\sqrt{3}}{2}$

So, $\cos\alpha = -\frac{\sqrt{3}}{2}$ and $\cos\beta = \frac{\sqrt{3}}{2}$

Hence, $\sec\alpha \times \sec\beta = \frac{1}{\cos\alpha \times \cos\beta} = -\frac{4}{3}$

95. (c) Given sin β is the HM of sin α and cos α

$\Rightarrow \sin\beta = \frac{2\sin\alpha\cos\alpha}{\sin\alpha + \cos\alpha}$ ...(i)

And sin θ is the AM of sin α and cos α

$\sin\theta = \frac{\sin\alpha + \cos\alpha}{2}$

Now, $\sqrt{2}\sin\left(\alpha + \frac{\pi}{4}\right)\sin\beta$

$= \sqrt{2}\left(\sin\alpha\cos\frac{\pi}{4} + \cos\alpha\sin\frac{\pi}{4}\right)\sin\beta$

(sin α + cos α) sin β

$= \frac{2\sin\alpha\cos\alpha}{\sin\beta}\sin\beta$ {From eq. (i)}

= sin 2α

And $\cos\left(\alpha - \frac{\pi}{4}\right) = \cos\alpha\cos\frac{\pi}{4} + \sin\alpha\sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}\{\cos\alpha + \sin\alpha\}$

$= \frac{1}{\sqrt{2}}\{2\sin\theta\}$ {From eq. (ii)}

$= \sqrt{2}\sin\theta$

Hence, both the statements are correct here.

96. (b) $m\tan(\theta - 30°) = n\tan(\theta + 120°)$

$\Rightarrow \frac{m}{n} = \frac{\tan(\theta + 120°)}{\tan(\theta - 30°)}$

or $\frac{m+n}{m-n}=\frac{\tan(\theta+120^\circ)+\tan(\theta-30^\circ)}{\tan(\theta+120^\circ)-\tan(\theta-30^\circ)}$

$$=\frac{\sin(\theta+120^\circ)\cos(\theta-30^\circ)+\sin(\theta-30^\circ)\cos(\theta+120^\circ)}{\sin(\theta+120^\circ)\cos(\theta-30^\circ)-\sin(\theta-30^\circ)\cos(\theta+120^\circ)}$$

$$\frac{\sin\{\theta+120^\circ+\theta-30^\circ\}}{\sin(\theta+120^\circ-\theta+30^\circ)}=\frac{\sin(90^\circ+2\theta)}{\sin 150^\circ}$$

$= 2\cos 2\theta$

97. (b) Let $A = \sin^2\theta + \cos^4\theta$

$\Rightarrow A = \cos^4\theta - \cos^2\theta + 1$

$$A=\left(\cos^2\theta-\frac{1}{2}\right)^2+\frac{3}{4}$$

$\Rightarrow A \ge \frac{3}{4}$ ...(i)

and $\sin^2\theta + \cos^4\theta = \sin^2\theta + \cos^2\theta.\cos^2\theta$

or $A \le \sin^2\theta + \cos^2\theta$

$\Rightarrow A \le 1$ ...(ii)

From eqs. (i) and (ii),

$\frac{3}{4} \le A \le 1$

98. (c) $\frac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}=\frac{1+\tan 9^\circ}{1-\tan 9^\circ}$

$= \tan(45^\circ + 9^\circ) = \tan 54^\circ$

99. (a) $\sin 2\theta = \cos 3\theta = \sin(90^\circ - 3\theta)$

$\Rightarrow 2\theta = 90^\circ - 3\theta$

or $5\theta = 90^\circ$ or $\theta = 18^\circ$

Then, $\sin\theta = \sin 18^\circ = \frac{\sqrt{5}-1}{4}$

100. (a) $\frac{\sin 34^\circ\cos 236^\circ-\sin 56^\circ\sin 124^\circ}{\cos 28^\circ\cos 88^\circ+\cos 178^\circ\sin 208^\circ}$

$$=\frac{\sin 34^\circ\cos(180^\circ+56^\circ)-\sin(90^\circ-34^\circ)\sin(180^\circ-56^\circ)}{\cos 28^\circ\cos(90^\circ-2^\circ)+\cos(180^\circ-2^\circ)\sin(180^\circ+28^\circ)}$$

$$=\frac{-\sin 34^\circ\cos 56^\circ-\cos 34^\circ\sin 56^\circ}{\cos 28^\circ\sin 2^\circ+\cos 2^\circ\sin 28^\circ}$$

$$=\frac{-\{\sin(34^\circ+56^\circ)\}}{\sin(28^\circ+2^\circ)}=-\frac{\sin 90^\circ}{\sin 30^\circ}=-2$$